Effective Quantum Dynamics of Quarks and "Gluons" in a Stochastic Background

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Abstract

The quantum dynamics of quarks and "gluons" and the scalar degrees of freedom associated with the non-linear regime of the non-linear gauge is derived. We discuss the subtleties in quantizing in a stochastic background. Then we show in detail that $\langle S_{YM}(t^a_\mu, \phi^a; \tilde{f}^a)\rangle_{\tilde{f}}$ only depends on t^a_μ, thus effectively proving that the scalars ϕ^a are non-propagating. Integrating out the scalars from $\langle S_{fermion}\rangle_{\tilde{f}}$ leads to fermion wave function that declines exponentially. Finally, we derive the effective action of the "gluons" and fermions resulting from stochastic averaging. We show that it leads to a confining four-fermi interaction.

I. INTRODUCTION

The non-linear gauge condition[1]

$$(\partial \cdot D^{ab})(\partial \cdot A^b) = (D^{ab} \cdot \partial)(\partial \cdot A^b) = (\partial^2 \delta^{ab} - g\epsilon^{abc} A^c_\mu \partial_\mu)(\partial \cdot A^b) = 0, \tag{1}$$

and its physical consequences has been discussed by this author in a number of papers. This gauge condition is a natural generalization in Yang-Mills theory of the Coulomb gauge in Abelian theory. The reasons for this claim are:

(a) In the Abelian limit, $(\partial \cdot D) \rightarrow \partial^2$ and since ∂^2 is positive definite, the gauge-condition yields the Coulomb gauge.

(b) The gauge condition has two regimes, the linear regime given by $\partial \cdot A^a = 0$, the Coulomb gauge, and the non-linear regime defined by $\partial \cdot A^a = f^a \neq 0$, which is also the zero mode of $\partial \cdot D$. Thus the non-linear regime corresponds to the "Gribov" horizon on the surface $\partial \cdot A^a = f^a(x)$.

(c) The linear and non-linear regimes do not mix in the sense that field configurations in the non-linear regime cannot be gauge transformed to the Coulomb gauge and vice versa.[2]

(d) If we consider the running of the coupling constant, we find that for the short-distance regime where $g \rightarrow 0$ (asymptotic freedom phase), the non-linear gauge reduces to $\partial^2(\partial \cdot A^a) = 0$, yielding the Coulomb gauge because ∂^2 is positive definite. This is consistent with the fact that transverse gluons are physical degrees of freedom at short-distances. However, as we increase the distance scale, g increases and before the running coupling becomes too large, where perturbation theory loses validity, the full non-linear character of the gauge condition becomes important where the relevant degrees of freedom are the scalars $f^a(x)$ and the "gluons" $t^a_\mu(x)$, which decompose the Yang-Mills potential[3] as

$$A^a_\mu = \frac{1}{(1 + \vec{f} \cdot \vec{f})}(\delta^{ab} + \epsilon^{abc} f^c + f^a f^b)(\frac{1}{g}\partial_\mu f^b + t^b_\mu). \tag{2}$$

This means that the non-linear gauge continuously interpolates short-distance and large distance physics with their corresponding degrees of freedom. Since we consider quantum effects in the running of the coupling, the non-linear gauge is a quantum gauge condition.

(e) Lastly, geometrically, configuration space analysis showed that non-linear gauge condition is a modification of the global orthogonal gauge condition, which does not exist in non-Abelian theories and gives the Coulomb gauge in Abelian theory.[4]

As for the physical consequences of the non-linear regime of the non-linear gauge, the following results had been established by the author:

(a) The Yang-Mills action is quartic in t_μ^a and infinitely non-linear in f^a. The pure f^a action has a $\frac{1}{g^2}$ factor and its kinetic term goes like $(\partial f^a)^4$. Clearly, these hint of non-perturbative physics.[5]

(b) The pure f^a dynamic shows that all spherically symmetric $\tilde{f}^a(x)$ with $x = (x_\mu x_\mu)^{1/2}$ are classical configurations with zero field strength. The pure f^a dynamics has a very broad minimum with zero action. Because of the infinite degeneracy in classical configurations, the author proposed to treat $\tilde{f}^a$ as a stochastic variable with a white-noise distribution. This resulted in the area law behaviour of the Wilson loop, which means a linear potential for static sources.[3]

(c) If the stochastic treatment of the classical configuration $\tilde{f}^a(x)$ yields a linear potential between static sources, full quantum dynamics of f^a shows equivalence to an $O(1,3)$ non-linear σ model in 2D.[6] The proof made use of the Parisi-Sourlas mechanism.[7] Since the σ model is $O(1,3)$ and not $O(4)$ resulting in a kinetic term with a wrong sign, the proof of confinement is purely formal. Furthermore, the proof of confinement only involves the scalars and not the "gluons" and quarks.

(d) When we consider the classical dynamics of the "gluons" in the spherically symmetric background $\tilde{f}^a(x)$, it was shown that stochastically averaging $\tilde{f}^a(x)$ yielded a mass for the t_μ^a and the loss of its self-interactions.[8] Thus, we have shown the mechanism for the mass gap. As for the loss of self-interactions, the result was also arrived at by Kondo.[9]

(e) Finally, the author proposed the concept of limited gauge invariance and showed that in both the linear and non-linear regimes of the non-linear gauge we can define potentials that are gauge-invariant within their limited context.[8]

In this paper, we will address the shortcomings noted in (c) by considering quarks and "gluons" in a stochastic background. We will decompose the scalar $f^a(x)$ via

$$f^a(x) = \tilde{f}^a(x) + \phi^a(x). \tag{3}$$

We will note the subtleties in treating quantum and stochastic fluctuations. We will derive the effective action of quarks and "gluons" by averaging over the stochastic background. Finally, we will derive a confining non-local four-fermi interaction.

II. THE ACTION IN THE NON-LINEAR GAUGE

We will consider $SU(2)$ theory with the following action

$$S = S_{YM} + S_{fermion} = \int d^4x\{\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}i\gamma_\mu D_\mu\psi\}. \tag{4}$$

Introducing the Fadeev-Popov trick of resolving unity using the gauge condition given by equation (1), we find the vacuum to vacuum functional

$$W(0) = \int (dA^a_\mu)(d\psi)(d\bar{\psi})det\theta\delta(\partial \cdot D(\partial \cdot A))e^{-S}, \tag{5}$$

where

$$\theta^{ad} = (D\cdot^{ab}\partial)(\partial \cdot D^{bd}) - g\epsilon^{abc}(\partial_\mu(\partial \cdot A^b))D^{cd}_\mu. \tag{6}$$

Note that in the linear regime, where the coupling g is very weak and thus the smallest eigenvalue of the positive definite ∂^2 (in R^4) is not lowered to zero, $(\partial \cdot D)$ is still positive definite and the path-integral given by equation (5) reduces to the transverse gauge path-integral.

As we further increase the coupling constant, we get to the Gribov horizon of $\partial \cdot A^a = 0$ surface. We are still on the Coulomb surface but the Fadeev-Popov operator is now singular. This regime had been discussed extensively in the late 1970's and 1980s. The conjecture is that restricting the transverse gauge path-integral to within the central Gribov region[10] or the fundamental modular region results in confinement[11]. This author has a different view: confinement happens in the next stage.

Further increasing the distance scale, we get to the regime where transverse gluons should no longer be relevant degrees. The reasons for this are the existence of the mass gap (massive vector fields are not transverse) and the fact that it is essentially impossible to get a confining interaction from the exchange of transverse gluons. Thus, we should be getting off the Coulomb surface and the non-linear gauge seems to be a natural gauge to consider. Here, we get to the Gribov horizon of the $\partial \cdot A^a = f^a \neq 0$ surface where the zero mode of $(\partial \cdot D)$ is $\partial \cdot A^a$ itself. However, even though $\partial \cdot D$ is singular, the corresponding operator θ given by equation (6) is non-singular[4]. The path-integral is still well-defined, presenting no need to deal with zero modes.

In this non-linear regime, we can decompose A^a_μ in terms of the scalars f^a and "gluons"

t^a_μ as given in equation (2). These new degrees of freedom satisfy the constraints

$$\partial \cdot t^a - \frac{1}{g\ell^2} f^a = 0 \tag{7}$$

$$\rho^a = \frac{1}{(1+\vec{f}\cdot\vec{f})^2}[\epsilon^{abc} + \epsilon^{abd} f^d f^c - \epsilon^{acd} f^d f^b + f^a f^d \epsilon^{dbc}$$
$$-f^a(1+\vec{f}\cdot\vec{f})\delta^{bc} - f^c(1+\vec{f}\cdot\vec{f})\delta^{ab}]\partial_\mu f^b t^c_\mu = 0, \tag{8}$$

giving the same number of degrees of freedom as the original $A^a_\mu(x)$. Substituting equation (2) in equation (4), we get

$$S = \int d^4x \{\langle \frac{1}{g^2} Z^2(f) + \frac{2}{g} Z(f)\cdot L(f,t) + [2Z(f)\cdot Q(f,t) + L(f,t)\cdot L(f,t)]$$
$$+2gL(f,t)\cdot Q(f,t) + g^2 Q(f,t)\cdot Q(f,t)\rangle + \langle \bar{\psi} i\gamma_\mu \partial_\mu \psi$$
$$-ig\bar{\psi}\gamma_\mu T^a \psi [R^{ab}(f)(\frac{1}{g}\partial_\mu f^b + t^b_\mu)]\rangle\}, \tag{9}$$

where

$$Z^a_{\mu\nu}(f) = X^{abc}(f)\partial_\mu f^b \partial_\nu f^c, \tag{10}$$

$$L^a_{\mu\nu}(f,t) = R^{ab}(f)(\partial_\mu t^b_\nu - \partial_\nu t^b_\mu) + Y^{abc}(\partial_\mu f^b t^c_\nu - \partial_\nu f^b t^c_\mu), \tag{11}$$

$$Q^a_{\mu\nu}(f,t) = T^{abc}(f) t^b_\mu t^c_\nu, \tag{12}$$

$$X^{abc}(f) = \frac{1}{(1+\vec{f}\cdot\vec{f})^2}[-(1+2\vec{f}\cdot\vec{f})\epsilon^{abc} + 2\delta^{ab} f^c - 2\delta^{ac} f^b$$
$$+3\epsilon^{abd} f^d f^c - 3\epsilon^{acd} f^d f^b + \epsilon^{bcd} f^a f^d], \tag{13}$$

$$R^{ab}(f) = \frac{1}{(1+\vec{f}\cdot\vec{f})}(\delta^{ab} + \epsilon^{abc} f^c + f^a f^b), \tag{14}$$

$$Y^{abc} = \frac{1}{(1+\vec{f}\cdot\vec{f})^2}[-(\vec{f}\cdot\vec{f})\epsilon^{abc} + (1+\vec{f}\cdot\vec{f}) f^a \delta^{bc} - (1-\vec{f}\cdot\vec{f})\delta^{ac} f^b$$
$$+3\epsilon^{cad} f^d f^b - 2f^a f^b f^c + \epsilon^{abd} f^d f^c + f^a \epsilon^{bcd} f^d], \tag{15}$$

$$T^{abc} = \frac{1}{(1+\vec{f}\cdot\vec{f})^2}[\epsilon^{abc} + (1+\vec{f}\cdot\vec{f}) f^b \delta^{ac} - (1+\vec{f}\cdot\vec{f}) f^c \delta^{ab}$$
$$+\epsilon^{abd} f^d f^c + f^a \epsilon^{bcd} f^d + \epsilon^{acd} f^d f^b]. \tag{16}$$

The pure f^a dynamics given by Z^2 has a class of classical configurations with zero field strength: the spherically symmetric $\tilde{f}^a(x)$, with $x = (x_\mu x_\nu)^{1/2}$ (we are in R^4). This follows from the anti-symmetry of X^{abc} with respect to the last two indices and that for spherically symmetric $\tilde{f}^a(x)$, $\partial_\mu \tilde{f}^a(x) = \frac{x_\mu}{x}\frac{d\tilde{f}^a}{dx}$. Since we will be expanding about a class of classical configurations, the author proposed to treat the spherically symmetric $\tilde{f}^a(x)$ stochastically with a white-noise distribution.

What will be done in the remainder of this section is to substitute the background decomposition given by equation (3) in the action given by equations (9) and (10) to (16). The resulting action is quantic in t^a_μ and infinitely non-linear in ϕ^a with coefficients that are functions of $\tilde{f}^a(x)$. The result is (see Appendix A).

$$\begin{aligned}
S &= \int d^4x\{[K^{ab}_{\mu\nu}(\tilde{f})\partial_\mu\phi^a\partial_\nu\phi^b + M^{ab}(\tilde{f})\phi^a\phi^b + N^{ab}_\mu(\tilde{f})\partial_\mu\phi^a\phi^b \\
&+ \sum_{n=3}^{\infty}\langle I^{a_1\cdots a_n}(\tilde{f})\phi^{a_1}\phi^{a_2} + J^{a_1\cdots a_n}(\tilde{f})\phi^{a_1}\partial_\mu\phi^{a_2} + H^{a_1\cdots a_n}\partial_\mu\phi^{a_1}\partial_\nu\phi^{a_2}\rangle \\
&\times \phi^{a_3}\cdots\phi^{a_n}] + \frac{1}{4}[\mathbb{R}^{ab}(\tilde{f})(\partial_\mu t^a_\nu - \partial_\nu t^a_\mu)(\partial_\mu t^b_\nu - \partial_\nu t^b_\mu) \\
&+ 2\mathbb{S}^{ab}(\tilde{f})(\partial_\mu t^a_\nu - \partial_\nu t^a_\mu)(\frac{x_\mu}{x}t^b_\nu - \frac{x_\nu}{x}t^b_\mu) + 2\mathbb{Y}^{ab}(\tilde{f})t^a_\mu t^b_\nu \\
&+ 2g\mathbb{U}^{abc}(\partial_\mu t^a_\nu - \partial_\nu t^a_\mu)t^b_\mu t^c_\nu + g^2\mathbb{T}^{abcd}t^a_\mu t^b_\mu t^c_\nu t^d_\nu] \\
&+ \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n!}\frac{\delta^n}{\delta f^{a_1}(x_2)\cdots\delta f^{a_n}(x_n)}[\frac{2}{g}Z(f)\cdot L(f,t) + L^2(f,t) + 2Z(f)\cdot Q(f,t) \\
&+ gL(f,t)\cdot Q(f,t) + g^2Q(f,t)Q(f,t)]_{f=\tilde{f}}\phi^{a_1}(x_1)\cdots\phi^{a_n}(x_n) \\
&+ [\bar{\psi}i\gamma_\mu\partial_\mu\psi + \bar{\psi}i\gamma_\mu T^a\langle R^{ab}(\tilde{f})(\frac{1}{g}\partial_\mu\tilde{f}^b + \frac{1}{g}\partial_\mu\phi^b + t^b_\mu) \\
&+ \sum_{n=1}^{\infty}\frac{1}{n!}\frac{\delta^n R^{ab}(\tilde{f})}{\delta f^{c_1}(x_1)\cdots\delta f^{c_n}(x_n)}(\frac{1}{g}\partial_\mu\tilde{f}^b + \frac{1}{g}\partial_\mu\phi^b + t^b_\mu)\phi^{c_1}(x_1)\cdots\phi^{c_n}(x_n)\rangle\psi(x)]\}. \quad (17)
\end{aligned}$$

The first term bracketted with [ ] comes from $Z^2(f)$ where we made use of $Z^a_{\mu\nu}(\tilde{f}) = 0$. The second bracketted terms have been discussed in a previous paper[9], where the author presented the classical dynamics of "gluons" in a stochastic vacuum. The third shows the interactions between the "gluons" and scalars ϕ^a. The fourth gives the fermion action, with the interaction between the fermions and "gluons" and scalars in a classical background $\tilde{f}^a(x)$. Equation (17) looks very complicated where the scalars are infinitely non-linear and the background $\tilde{f}^a(x)$ is rather involved. However, this complicated action is dramatically simplified upon stochastic averaging.

Introduce the white noise distribution

$$P[\tilde{f}] = \mathcal{N}exp.\{-\frac{1}{\ell}\int_0^\infty \tilde{f}^a(s)\tilde{f}^a(s)ds\} \quad (18)$$

where ℓ is the scale when non-perturbative physics is important. From the running of the coupling, we must have $\ell \sim \Lambda^{-1}_{QCD}$. When we average equation (17) using equation (18), a tremendous simplification results (see Appendix B)

$$\langle S\rangle_{\tilde{f}} = \int(d\tilde{f}^a(x))S(t^a_\mu,\phi^a,\psi,\bar{\psi};\tilde{f})P[\tilde{f}]$$

$$= \frac{1}{4}\int d^4x\{\frac{1}{3}(\partial_\mu t^a_\nu - \partial_\nu t^a_\mu)^2 + \frac{3}{2}(\frac{n}{\ell})^2 t^a_\mu t^a_\mu + \bar{\psi} i\gamma_\mu[\partial_\mu - gT^a(\frac{1}{3}t^a_\mu$$
$$+ \frac{1}{3g}\partial_\mu\phi^a - \frac{2}{3g}(\frac{n}{\ell})\frac{x_\mu}{x}\phi^a)]\psi\}. \tag{19}$$

From equation (19), we see that the scalars ϕ^a do not have a kinetic term; they do not propagate.

When we get the equation of motion for ϕ^a, we find

$$\partial_\mu\psi + \frac{1}{(\frac{\ell}{n})}g^2\frac{x_\mu}{x}\psi = 0. \tag{20}$$

The spherically symmetric solution of equation (20) is

$$\psi(x) = \psi_0 exp.\{-\frac{1}{(\frac{\ell}{n})}g^2 x\}, \tag{21}$$

which is an exponentially vanishing fermion field with effective length $= \frac{\ell}{ng^2} \sim \Lambda^{-1}_{QCD}$. This behaviour is a signal of confinement.

III. QUANTUM FIELD THEORY IN A STOCHASTIC BACKGROUND

We would like to quantize $t^a_\mu, \psi, \bar{\psi}$ and ϕ^a in the presence of the stochastic background $\tilde{f}^a(x)$. In essence, we need to deal with quantum fluctuations on top of classical stochastic variables. This poses an ambiguity due to the ordering of how these two fluctuations are taken into account. One sequence is to consider quantum field theory in a stochastic background and then do the stochastic averaging. The other sequence is first to do the stochastic averaging on the classical action and then quantize the resulting theory.

Let us collectively represent the fields $t^a_\mu, \psi, \bar{\psi}$ and ϕ^a by the field Φ. The stochastic classical background is still given as $\tilde{f}$. We write the action of Φ in a background $\tilde{f}$, which includes gauge-fixing and the "Fadeev-Popov" determinant as $S[\Phi, \tilde{f}] = \int d^4x \mathcal{L}(\Phi, \tilde{f})$. Define

$$\langle S[\Phi, \tilde{f}\rangle_{\tilde{f}} \equiv \int (d\tilde{f})\mathcal{N} e^{-\frac{1}{\ell}\int_0^\infty \tilde{f}(s)\tilde{f}(s)} S[\Phi, \tilde{f}]$$
$$= S_{eff}[\Phi]. \tag{22}$$

The complete generating functional from which we compute n-point functions is

$$W_{eff}[J] = \int (d\Phi) e^{-S_{eff}[\Phi] - \int d^4x J\Phi} \tag{23}$$

From this generating functional, we compute the n-point function by

$$G_n(x_1,\cdots,x_n)=\frac{\delta^n W_{eff}[J]}{\delta J(x_1)\cdots\delta J(x_n)}. \tag{24}$$

On the other hand, we can first consider the complete generating functional in the presence of the background $\tilde{f}$. This is

$$W[J;\tilde{f}]=\int(d\Phi)e^{-S[\Phi,\tilde{f}]-\int d^4xJ\Phi}. \tag{25}$$

Doing stochastic averaging, we find

$$\begin{aligned} W'_{eff}[J] &= \langle W[J,\tilde{f}]\rangle_{\tilde{f}} \\ &= \int(d\Phi)\langle e^{-S[\Phi,\tilde{f}]}\rangle_{\tilde{f}}e^{-\int d^4xJ\Phi} \end{aligned} \tag{26}$$

Expanding the exponential and taking the stochastic average of each term, we find

$$\langle e^{-S[\Phi,\tilde{f}]}\rangle_{\tilde{f}}=e^{-S'_{eff}[\Phi]}, \tag{27}$$

where

$$\begin{aligned} S'_{eff}[\Phi] &= \langle S[\Phi,\tilde{f}]\rangle_{\tilde{f}}-\frac{1}{2}\int d^4xd^4y\langle\underbrace{\mathcal{L}(\Phi,\tilde{f};x)\mathcal{L}(\Phi,\tilde{f};y)}\rangle \\ &- \frac{1}{4!}\int d^4xd^4yd^4zd^4r\langle\underbrace{\mathcal{L}(\Phi,\tilde{f};x)\mathcal{L}(\Phi,\tilde{f};y)\mathcal{L}(\Phi,\tilde{f};z)\mathcal{L}(\Phi,\tilde{f};r)}\rangle_{\tilde{f}} \\ &+ \cdots \end{aligned} \tag{28}$$

The terms $\underbrace{\mathcal{L}(\Phi,\tilde{f};x)\mathcal{L}(\Phi,\tilde{f};y)}$, etc., are correlated points, which arise because the derivative of the white noise $\tilde{f}(x)$ is "smoothened-out" via

$$\frac{d\tilde{f}^a}{dx}=\frac{\tilde{f}^a(x+\frac{\ell}{n})-\tilde{f}^a(x)}{\frac{\ell}{n}} \tag{29}$$

Using equations (27) and (28) in (26), we find that $W'_{eff}[J]\neq W_{eff}[J]$. The n-point function before the stochastic averaging is given by

$$G_n(x_1\cdots,x_n;\tilde{f})=\frac{\delta^n W[J;\tilde{f}]}{\delta J(x_1)\cdots\delta J(x_n)}. \tag{30}$$

Obviously,

$$\begin{aligned} \langle G_n(x_1,\cdots,x_n;\tilde{f})\rangle_{\tilde{f}} &= \langle\frac{\delta^n W[J;\tilde{f}]}{\delta J(x_1)\cdots\delta J(x_n)}\rangle_{\tilde{f}} \\ &= \frac{\delta^n W'_{eff}[J]}{\delta J(x_1)\cdots\delta J(x_n)} \\ &= G'_n(x_1,\cdots,x_n) \end{aligned} \tag{31}$$

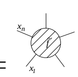
However, it is clear that the n-point function that appear in equation (31) is not equal to the n-point function given in equation (24).

Thus, we see that the method of first stochastically averaging the classical action in the presence of the background $\tilde{f}$ and then quantizing the theory is not the same as one that first quantizes theory in the presence of $\tilde{f}$ (see equation (30)) and then does the stochastic averaging.

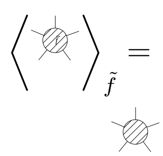
There is another subtlety in quantizing the theory even within the framework of doing the stochastic averaging only after quantizing the theory in the presence of $\tilde{f}$. Let us begin with equation (25), which represents the complete generating functional in the presence of the stochastic background $\tilde{f}$. This is diagrammatically represented by

$$W[J, \tilde{f}] = \tag{32}$$


while the n-point function given by equation (30) is represented by

$$G_n(x_1 \cdots, x_n; \tilde{f}) = \tag{33}$$


Equation (31) says that


$$\tag{34}$$

where the RHS of this diagrammatic expression represents the full n-point function from $W'_{eff}[J]$.

Following the usual field theory prescription, we define the connected Green function generating functional via

$$W[J; \tilde{f}] = e^{iZ[J,\tilde{f}]} \tag{35}$$

Diagrammatically, this is represented by


$$= Z[J; \tilde{f}]. \tag{36}$$

We compute the connected n-point Green function via

$$G^c_n(x_1, \cdots, x_n; \tilde{f}) = \frac{\delta^n Z[J; \tilde{f}]}{\delta J(x_1) \cdots \delta J(x_n)} \tag{37}$$

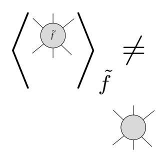
It is clear from equation (34) that

$$\begin{aligned} \langle Z[J;\tilde{f}]\rangle_{\tilde{f}} &= \frac{1}{i}\langle \ell n W[J;\tilde{f}]\rangle_{\tilde{f}} \\ &\neq \frac{1}{i}\ell n W'_{eff}[J] = Z'_{eff}[J] \end{aligned} \tag{38}$$

where the last line of equation (38) follows from equation (26). From equation (37), it follows that

$$\langle G^c_n(x_1,\cdots,x_n;\tilde{f})\rangle_{\tilde{f}} \neq G'^c_{eff,n}(x_1,\cdots,x_n), \tag{39}$$

where the RHS is evaluated using equation (26). Diagrammatically, this is represented by



(40)

If this non-equivalence is true for the connected n-point function it is easy to show that it is also true for the one-particle-irreducible functions derived from the $\Gamma[\tilde{\Phi},\tilde{f}]$ and $\Gamma[\tilde{\Phi}]$, which are derived via Legendre transformation. Define the "classical" fields

$$\tilde{\Phi}(\tilde{f}) = \frac{\delta Z[J;\tilde{f}]}{\delta J(x)}, \tag{41}$$

$$\tilde{\Phi}_{eff} = \frac{\delta Z'_{eff}[J]}{\delta J(x)}, \tag{42}$$

where equation (42) makes use of equations (38) and (26) and it is obvious that

$$\langle \tilde{\Phi}(\tilde{f})\rangle_{\tilde{f}} \neq \tilde{\Phi}_{eff}.$$

The effective action, which is also the one-particle-irreducible generating functional is defined by

$$\Gamma[\tilde{\Phi},\tilde{f}] = Z[J,\tilde{f}] - \int d^4x \tilde{\Phi}(\tilde{f})J, \tag{43}$$

$$\tilde{\Gamma}'_{eff}[\tilde{\Phi}_{eff}] = Z'_{eff}[J] - \int d^4x J\tilde{\Phi}_{eff}. \tag{44}$$

where it is implied that equations (41) and (42) are inverted to solve for the sources in terms of the "classical" fields. Again, it is clear that

$$\langle \tilde{\Gamma}[\tilde{\Phi},\tilde{f}]\rangle_{\tilde{f}} \neq \tilde{\Gamma}'_{eff}[\Phi_{eff}]. \tag{45}$$

Here, we see that although there is ambiguity when stochastic averaging is done at the level of connected and one-particle-irreducible generating functionals and n-point functions, there is no such ambiguity at the level of the full generating functional and full n-point functions. Since the S-matrix is expressed in terms of the full n-point functions, the quantum theory in a stochastic background is consistent and well-defined as long as we make use of the action given by equation (28).

IV. THE EFFECTIVE QUANTUM DYNAMICS

We begin with the path-integral in the non-linear gauge given by

$$\begin{aligned} W &= \int (dt^a_\mu)(df^a)(d\psi)(d\bar{\psi})\delta(\partial\cdot t^a - \frac{1}{g\ell^2}f^a)\delta(\rho^a) \\ &\cdot \; det^{-4}(1+\vec{f}\cdot\vec{f})det\theta exp.\{-(S_{YM}+S_{fermion})\}. \end{aligned} \tag{46}$$

Let us rewrite the delta functionals by

$$\delta(\partial\cdot t^a - \frac{1}{g\ell^2}f^a) = det[\frac{1}{(1+\vec{f}\cdot\vec{f})^j}]\delta(\frac{1}{(1+\vec{f}\cdot\vec{f})^j}(\partial\cdot t^a - \frac{1}{g\ell^2}f^a)) \tag{47}$$

$$\delta(\rho^a) = det[\frac{1}{(1+\vec{f}\cdot\vec{f})^k}]\delta(\frac{1}{(1+\vec{f}\cdot\vec{f})^k}\rho^a) \tag{48}$$

where j and k are positive integers. The reason for equations (47) and (48) will become clear later. Expressing the determinants in terms of ghosts, the path-integral can be written as

$$W = \int (dt^a_\mu)(df^a)(d\psi)(d\bar{\psi})(du^a)(d\bar{u}^a)exp.\{-S'\}, \tag{49}$$

where

$$S' = S_{YM} + S_{fermion} + S_{gf} + S_{ghosts}, \tag{50}$$

$$S_{gf} = \int d^4x\{(\frac{1}{\alpha})\frac{1}{(1+\vec{f}\cdot\vec{f})^{2j}}(\partial\cdot t^a - \frac{1}{g\ell^2}f^a)^2 + \frac{1}{\beta}\frac{1}{(1+\vec{f}\cdot\vec{f})^{2k}}(\rho^a)^2\}, \tag{51}$$

$$S_{ghosts} = \int d^4x\bar{u}^a\frac{1}{(1+\vec{f}\cdot\vec{f})^{4+j+k}}\theta^{ab}u^b. \tag{52}$$

Implementing the background decomposition given by equation (3) in the path-integral, we get the vacuum to vacuum functional in the background $\tilde{f}^a(x)$, i.e.,

$$W[\tilde{f}^a] = \int (dt^a_\mu)(d\phi^a)(d\psi)(d\bar{\psi})(du^a d\bar{u}^a)exp.\{-S'\}, \tag{53}$$

where $S'(t^a_\mu, \psi, \bar{\psi}, f^a = \tilde{f}^a + \phi^a, \bar{u}^a, u^a)$. As equations (26), (27), (28) and (34) show, as far as the full generating functional and the n-point Greens functions are concerned, replacing S' by S'_{eff} (see equation (28)) in equation (53) is equivalent to computing physical processes with $W[\tilde{f}^a]$ and then doing the stochastic averages at the end. Now we will see the reason for equations (47) and (48). Equation (28) says we have to evaluate

$$\langle S' \rangle_{\tilde{f}} = \langle S_{YM} \rangle + \langle S_{fermion} \rangle + \langle S_{gf} \rangle + \langle S_{ghosts} \rangle \quad (54)$$

$$\int d^4x d^4y \langle \underbrace{(\mathcal{L}_{YM} + \mathcal{L}_{fermion} + \mathcal{L}_{gf} + \mathcal{L}_{ghosts})_x (\mathcal{L}_{YM} + \mathcal{L}_{fermion} + \mathcal{L}_{gf} + \mathcal{L}_{ghosts}})_y \rangle_{\tilde{f}}, \quad (55)$$

etc.

The stochastic averages involve integrals of the form

$$\lim_{\sigma \to 0} (\pi^{-3/2} \sigma^{+3/2}) \int_0^\infty \frac{r^{2m}}{(1+r^2)^n} e^{-\sigma r^2} dr = \begin{cases} 0, & \text{for } m \le n \\ non-zero, finite, & \text{for } m = n+1 \\ diverges, & \text{for } m \ge 0, n+2. \end{cases} \quad (56)$$

Because of the $\frac{1}{(1+\vec{f} \cdot \vec{f})^{2j}}$ term that goes with $(\partial \cdot t^a - \frac{1}{g\ell^2} f^a)^2$, when we expand using the decomposition given by equation (3) and by suitable choice of j, the resulting integral will always yield zero. The same thing is true with the term $\sim (\rho^2)$ and the ghost term. Thus, $\langle S_{gf} \rangle_{\tilde{f}} = \langle S_{ghosts} \rangle_{\tilde{f}} = 0$. And $\langle S' \rangle_{\tilde{f}}$ yields just the term given by equation (19).

As for equation (55), all the correlated terms involving $\mathcal{L}_{gf}$ and $\mathcal{L}_{ghosts}$ vanish for the same reason as above. This means that S'_{eff} does not involve any ghosts $\bar{u}^a, u^a$ and we can just lump the ghosts measure with the normalization of the path-integral.

The fermion-fermion correlated term will yield the following non-local, four fermi interaction

$$NLFF = \frac{g^2}{2} \int d^4x d^4y (\bar{\psi} \gamma_\mu T^a \psi)_x \langle A^a_\mu(x) A^{a'}_\nu(y) \rangle_{\tilde{f}} (\psi \gamma_\nu \bar{T^{a'}} \psi)_y \quad (57)$$

We will evaluate the stochastic average by making use of

$$\partial_\mu A^a_\mu(x) = \frac{1}{g\ell^2} f^a(x) = \frac{1}{g\ell^2} \tilde{f}^a(x) + \frac{1}{g\ell^2} \phi^a(x). \quad (58)$$

From equation (58), we must have

$$\partial^x_\mu \partial^y_\nu \langle A^a_\mu(x) A^b_\nu(y) \rangle_{\tilde{f}} = \frac{1}{g^2 \ell^3} \delta(x-y) + ..., \quad (59)$$

where $x = (x_\mu x_\mu)^{1/2}$ and $y = (y_\mu y_\mu)^{1/2}$. Equation (59) implies that

$$\langle A_\mu^a(x) A_\nu^b(y)\rangle = (\frac{1}{g^2\ell^3})\frac{x_\mu}{x}\frac{y_\nu}{y}\delta^{ab}|x-y| + ... \tag{60}$$

The equivalence follows from the fact that for a spherically symmetric function, $\partial_\mu^x = \frac{x_\mu}{x}\frac{d}{dx}$ and $\frac{d^2}{dx^2}|x-y| = \delta(x-y)$. Substituting equation (60) in NLFF, we find

$$NLFF = \frac{1}{2}(\frac{1}{\ell^3})\int d^4x d^4y (\bar{\psi}\eta_\mu\gamma_\mu T^a\psi)_x |x-y| (\bar{\psi}\gamma_\nu\eta_\nu T^a\psi)_y + ... \tag{61}$$

where

$$\eta_\mu = (sin\theta_1 sin\theta_2 sin\phi, sin\theta_1 sin\theta_2 cos\phi, sin\theta_1 cos\theta_2, cos\theta_1), \tag{62}$$

i.e., η_μ represents the unit vectors in 4D spherical coordinates. If the fermion field is spherically symmetric, the angular integration does not vanish only when $\eta_\mu(\vec{x}) = \pm\eta_\mu(\vec{y})$, i.e., the 4D vectors are collinear. Using

$$\int d\Omega_4 \eta_\mu\eta_\nu = \frac{\pi^2}{2}\delta_{\mu\nu} \tag{63}$$

and $\int d\Omega_4 = \int_0^{2\pi}\int_0^\pi\int^\pi \sin^2\theta_1 \sin\theta_2 d\theta_1 d\theta_2 d\phi = 2\pi^2$, we find that we can write the equation (61) as

$$NLFF = \frac{1}{8}(\frac{1}{\ell^3})\int d^4x d^4y (\bar{\psi}\gamma_\mu T^a\psi)_x |\vec{x}-\vec{y}| (\bar{\psi}\gamma_\mu T^a\psi)_y + \cdots \tag{64}$$

where the points $\vec{x}$ and $\vec{y}$ in R^4 must be co-linear, suggesting "flux-tube" configuration. This is surprising because the mechanism involves spherically symmetric, stochastic $\tilde{f}^a(x)$ yet the four-fermi interaction yields a co-linear configuration with linear interaction. Furthermore, if we neglect the extra terms in equation (64) (which will involve quadratic terms in ϕ^a), the ϕ^a integral will yield the following delta functional

$$\delta(\partial_\mu(\bar{\psi}\gamma_\mu T^a\psi) - \frac{2g}{\ell/n}\frac{x_\mu}{x}(\bar{\psi}\gamma_\mu T^a\psi)). \tag{65}$$

As already discussed in section II, this implies equation (20) yielding fermion solutions given by equation (21), which points to fermions with effective length $\sim \Lambda_{QCD}^{-1}$ and with a linear potential as given by equation (64). This definitely shows quark confinement and that when we try to pull a fermion from inside a hadron, a colorless quark-anti quark pair (a meson), will be formed because of the linear potential. This is the picture of "string" breaking, although a string was never invoked in our mechanism (spherically symmetric, stochastic $\tilde{f}^a(x)$).

In summary, the effective theory for quarks and "gluons" under the approximations we make, is given by the path-integral

$$\begin{aligned} W &= \int (dt^a_\mu)(d\psi)(d\bar{\psi})\delta(\partial_\mu(\bar{\psi}\gamma_\mu T^a\psi) - \frac{2g}{\ell/n}\frac{x_\mu}{x}(\bar{\psi}\gamma_\mu T^a\psi)) \\ &\times \; exp.\{-S_{eff}(t^a_\mu,\psi,\bar{\psi})\} \end{aligned} \tag{66}$$

where

$$\begin{aligned} S_{eff}(t^a_\mu,\psi,\bar{\psi}) &= \int d^4x\{\frac{1}{12}(\partial_\mu t^a_\nu - \partial_\nu t^a_\mu)^2 \\ &+ \frac{3}{6}(\frac{n}{\ell})^2 t^a_\mu t^a_\mu + \bar{\psi}i\gamma_\mu\partial_\mu\psi - \frac{ig}{3}(\bar{\psi}\gamma_\mu T^a\psi)t^a_\mu\} \\ &+ \frac{1}{8}(\frac{1}{\ell^3})\int d^4x d^4y(\bar{\psi}\gamma_\mu T^a\psi)_x|\vec{x}-\vec{y}|(\bar{\psi}\gamma_\mu T^a\psi)_y. \end{aligned} \tag{67}$$

V. CONCLUSION

We have derived an effective dynamics for quarks and "gluons" as given in equations (66) and (67). The effective action clearly shows a mass gap, i.e., the "gluons" acquired a mass and the quarks are confined. The mechanism for all these is the spherically symmetric $\tilde{f}^a(x)$ treated as a stochastic variable. These vacuum configurations arise from the non-linear regime of the non-linear gauge, which we claim is the natural generalization in Yang-Mills theory of the Coulomb gauge in Abelian theory.

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[1] J. A. Magpantay, Progress of Theoretical Physics **91**, 573 (1994).

[2] J. A. Magpantay and E. C. Cuansing, Modern Physics Letters **A11**, 87 (1996).

[3] J. A. Magpantay, Modern Physics Letters **A14**, 442 (1999).

[4] J. A. Magpantay, hep-th/0110038 **version 2** (11 July 2003).

[5] J. A. Magpantay, *in Mathematical Methods of Quantum Physics: Essays in honor of Hiroshi Ezawa, Ed. C.C. Bernido, et al* (Gordon and Breach Science, 1999).

[6] J. A. Magpantay, International Journal of Modern Physics **A15**, 1613 (2000).

[7] G. Parisi and N. Sourlas, Phys. Rev. Lett **43**, 744 (1979).

[8] J. A. Magpantay, hep-th/0203178 **version 2** (11 July 2003).

[9] K. Kondo, hep-th/0311033 **version 2** (23 Jan. 2004).

[10] V. Gribov, Nuclear Physics **B139**, 1 (1978).

[11] D. Zwanziger, Nuclear Physics **B518**, 237 (1998).

APPENDIX A

Here, we will derive equation (17). The starting point is equation (9). Using equation (3) in equations (10) to (16), we get

$$\begin{aligned} Z^a_{\mu\nu}(f=\tilde{f}+\phi) &= X^{abc}(\tilde{f}+\phi)(\partial_\mu\tilde{f}^b+\partial_\mu\phi^c)(\partial_\nu\tilde{f}^c+\partial_\nu\phi^c) \\ &= [X^{abc}(\tilde{f})+\frac{\delta X^{abc}}{\delta f^d}(\tilde{f})\phi^d+\frac{1}{2!}\frac{\delta^2 X^{abc}}{\delta f^d\delta f^e}\phi^d\phi^e+\cdots] \\ &\times\ (\partial_\mu\tilde{f}^b+\partial_\mu\phi^b)(\partial_\nu\tilde{f}^c+\partial_\nu\phi^c) \end{aligned} \quad (A1)$$

Since $X^{abc}(\tilde{f})\partial_\mu\tilde{f}^b\partial_\nu\tilde{f}^c=0$, we find that

$$K^{ab}_{\mu\nu}(\tilde{f}) = 2X^{cda}(\tilde{f})X^{ceb}(\tilde{f})\partial_\alpha\tilde{f}^d\partial_\alpha\tilde{f}^e\delta_{\mu\nu}-2X^{cda}(\tilde{f})X^{ceb}(\tilde{f})\partial_\mu\tilde{f}^d\partial_\nu\tilde{f}^e \quad (A2)$$

$$M^{ab}(\tilde{f}) = \frac{\delta X^{cde}}{\delta f^a}(\tilde{f})\frac{\delta X^{cfg}}{\delta f^b}(\tilde{f})\partial_\mu\tilde{f}^d\partial_\mu\tilde{f}^f\partial_\nu\tilde{f}^e\partial_\nu\tilde{f}^g \quad (A3)$$

$$N^{ab}_\nu(\tilde{f}) = 4X^{cda}\frac{\delta X^{cef}}{\delta f^b}\partial_\nu\tilde{f}^e\partial_\nu\tilde{f}^d\partial_\mu\tilde{f}^f \quad (A4)$$

The rest of the terms in the first bracket of equation (17) can be arrived at by considering the second and higher functional derivatives of X^{abc}

As for the second bracketted terms in equation (17), these had been derived in reference [8]. The third bracketted terms, which give the scalars-"gluons" interaction in a background $\tilde{f}^a$, makes use of equations (10) to (16). This is an infinite series of complicated terms. The important point is that each term has sufficient powers of $(1+\vec{f}\cdot\vec{f})$ in the denominator, which makes it vanish upon stochastic averaging.

As for the fourth bracketted terms, these arise from

$$\begin{aligned} A^a_\mu(x) &= R^{ab}(f)(\frac{1}{g}\partial_\mu f^b+t^b_\mu) \\ &= R^{ab}(\tilde{f}+\phi)(\frac{1}{g}\partial_\mu\tilde{f}^b+\frac{1}{g}\partial_\mu\phi^b+t^b_\mu) \\ &= [R^{ab}(\tilde{f})+\frac{\delta R^{ab}}{\delta f^c}(\tilde{f})\phi^c+\frac{1}{2!}\frac{\delta^2 R^{ab}}{\delta f^c\delta f^d}(\tilde{f})\phi^c\phi^d+\cdots] \\ &\quad (\frac{1}{g}\partial_\mu\tilde{f}^b+\frac{1}{g}\partial_\mu\phi^b+t^b_\mu). \end{aligned} \quad (A5)$$

APPENDIX B

We will derive equation (19) from equation (17). We will make use of equation (18) and (56). Let us evaluate in detail one stochastic average.

$$\langle K^{ab}_{\mu\nu}(\tilde{f})\rangle_{\tilde{f}} \;=\; \mathcal{N}\int(d\tilde{f}^a)X^{cda}(\tilde{f})X^{ceb}(\tilde{f})(\partial_\alpha\tilde{f}^d\partial_\alpha\tilde{f}^e\delta_{\mu\nu}-\partial_\mu\tilde{f}^d\partial_\nu\tilde{f}^e)$$
$$\times\;\; exp.\{-\frac{1}{\ell}\int_0^\infty ds\tilde{f}^a(s)\tilde{f}^a(s)\}. \quad \text{(B1)}$$

Using equation (29) and equation (13), we find that we need

$$\langle\tilde{f}^d(x+\frac{\ell}{n})\rangle_{\tilde{f}} \;=\; 0 \quad \text{(B2)}$$
$$\langle\tilde{f}^d(x+\frac{\ell}{n})_{\tilde{f}}(x+\frac{\ell}{n})\rangle_{\tilde{f}} \;=\; \frac{1}{2}\delta^{de}\lim_{\sigma\longrightarrow 0}(\frac{1}{\sigma}) \quad \text{(B3)}$$

where $\sigma=\frac{\Delta s}{\ell}$. The stochastic average at point x, which goes with $\frac{1}{\sigma}$ is of the form

$$\pi^{-3/2}\sigma^{3/2}\int\frac{r^2drd\Omega}{(1+r^2)^4}[-(1+2r^2)\epsilon^{cda}+2\delta^{ad}x^e-2\delta^{ce}x^d$$
$$+3\epsilon^{cdf}x^fx^a-3\epsilon^{caf}x^fx^d+\epsilon^{daf}x^cx^f][-(1+2r^2)\epsilon^{ceb}$$
$$+2\delta^{ce}x^b-2\delta^{cb}x^e+3\epsilon^{ceg}x^gx^b-3\epsilon^{cbg}x^gx^e+\epsilon^{ebg}x^gx^c]$$
$$\times e^{-\sigma r^2} \quad \text{(B4)}$$

Since the numerator only has 6 powers of r at most, while the denominator has 8, the integral is at best

$$\pi^{-3/2}\sigma^{+3/2}\int_0^\infty\frac{e^{-\sigma r^2}}{1+r^2}dr=\pi^{-3/2}\sigma^{-3/2}[1-\Phi(\sigma^{3/2})\frac{\pi}{2}e^{\sigma}] \quad \text{(B5)}$$

where Φ is the error function (not to be confused with the field that collectively represents $t^a_\mu,\phi^a,\phi,\bar{\psi}$ in section III) with expansion

$$\Phi(\sigma^{1/2})=\frac{2}{\sqrt{\pi}}[\sigma^{1/2}-\frac{1}{3}\sigma^{3/2}+\frac{1}{10}\sigma^{5/2}+\cdots] \quad \text{(B6)}$$

Thus, this integral behaves, at best, like $\sigma^{+3/2}$, as $\sigma\longrightarrow 0$. Combining this with the $\frac{1}{\sigma}$ factor, we find this term to vanish like $\sigma^{1/2}$. There is another term in $\langle K^{ab}_{\mu\nu}\rangle_{\tilde{f}}$ with $\tilde{f}^d(x)\tilde{f}^e(x)$ of the derivatives, which will combine with the rest given in (B.4). But this term will yield a stochastic average which will vanish like σ as $\sigma\longrightarrow 0$. Thus, we find

$$\langle K^{ab}_{\mu\nu}(\tilde{f})\rangle_{\tilde{f}}=0. \quad \text{(B7)}$$

Using similar analysis, it is trivial to show that all the other stochastic averages in the first and 3rd brackets of equation (17) vanish. The only non-vanishing stochastic averages are

$$\langle \mathbb{R}^{ab}(\tilde{f}) \rangle_{\tilde{f}} = \frac{1}{3}\delta^{ab} \tag{B8}$$

$$\langle \mathbb{Y}^{ab} \rangle_{\tilde{f}} = \delta^{ab}(\frac{3}{4})(\frac{n^2}{\ell^2}) \tag{B9}$$

These averages yield the first and second terms of equation (19).

Finally, from equation (A.5), we get

$$\langle A^a_\mu \rangle_{\tilde{f}} = \frac{1}{3}t^a_\mu + \frac{1}{3g}\partial_\mu \phi^a - \frac{2}{3g}(\frac{n}{\ell})\frac{x_\mu}{x}\phi^a. \tag{B10}$$